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# The application of loss to the helix support structure of a traveling wave tube

Torbert, John Hallett

Monterey, California: U.S. Naval Postgraduate School

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THE APPLICATION OF LOSS TO THE  
HELIX SUPPORT STRUCTURE OF A  
TRAVELING WAVE TUBE

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JOHN H. TORBERT

1953

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STRUCTURE OF A TRAVELING WAVE TUBE

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THE APPLICATION OF LOSS TO THE HELIX SUPPORT  
STRUCTURE OF A TRAVELING WAVE TUBE

by

John Hallett Torbert  
Lieutenant, United States Navy

Submitted in partial fulfillment  
of the requirements for  
CERTIFICATE OF COMPLETION

United States Naval Postgraduate School  
Monterey, California  
1953

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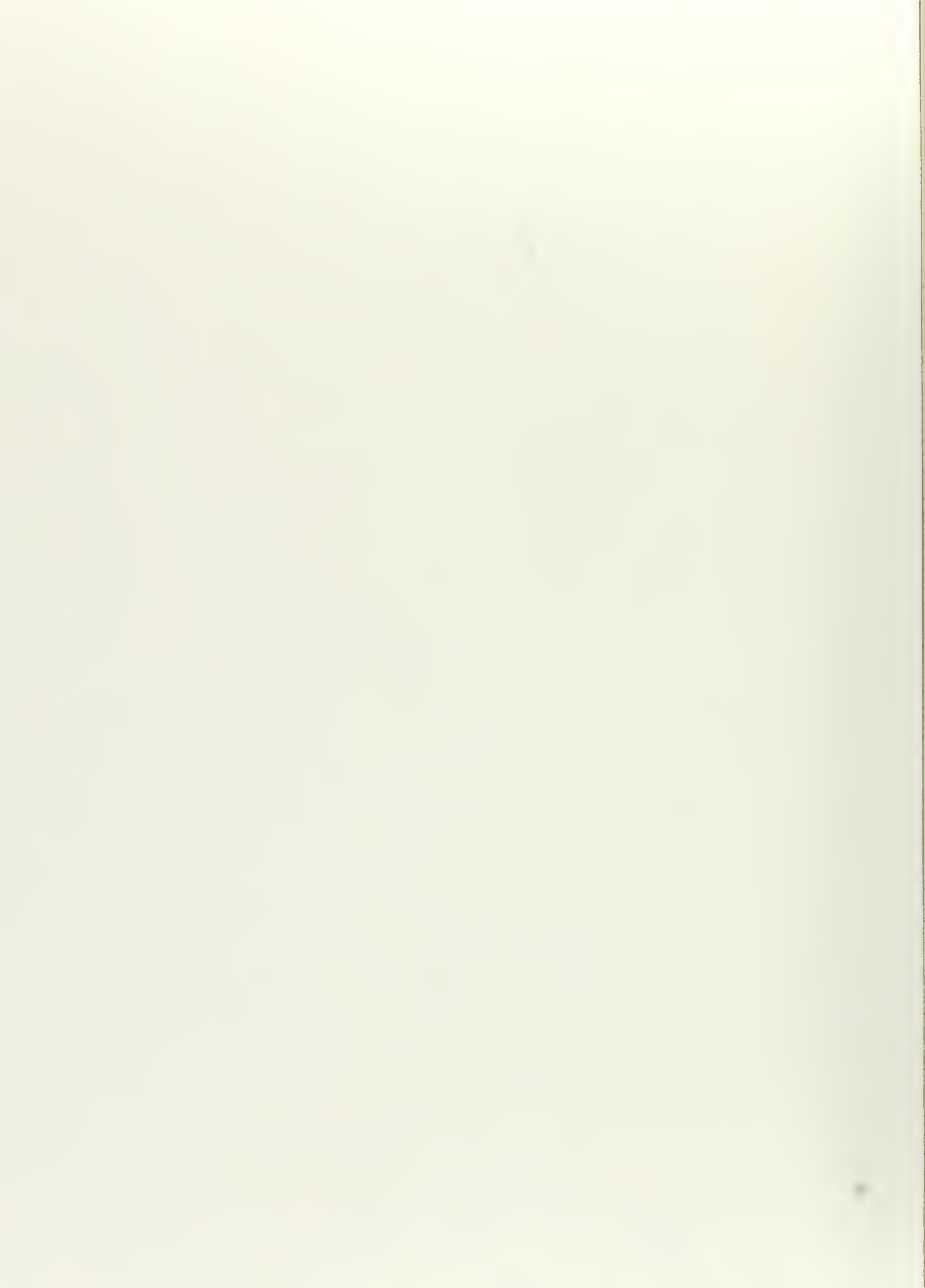
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the thesis requirements for  
CERTIFICATE OF COMPLETION

from the  
United States Naval Postgraduate School



## PREFACE

I wish to express my sincere appreciation to the Federal Telecommunication Laboratories, Inc. and Admiral C. F. Holden, USN (Ret.), President for the opportunity to work with them on some of the problems of their traveling wave tube program.

I would especially like to thank the members of the F. T. L. Tube Department, Mr. A. K. Wing, Jr. (Dept. Head), Mr. R. E. White and Mr. J. H. Bryant for their assistance and encouragement in learning the little I did of the tube business.

My thanks, also, is expressed to the engineers with whom I worked and whose valuable know-how and technical assistance was extended to me.



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# TABLE OF SYMBOLS & ABBREVIATIONS

TWT	=	Traveling Wave Tube
A	=	$20 \log 10^{1/3}$
a	=	Attenuation constant - radius of helix
B	=	Shunt susceptance
C	=	$\left(\frac{KI_0}{4V_0}\right)^{1/3} \approx 0.02$
E	=	Electric field
e	=	Charge of electron
G	=	Gain
Ge	=	Electronic gain
H	=	Magnetic field
I	=	Current
I <sub>0</sub>	=	Average electron convection current
I <sub>0</sub> , I <sub>1</sub> , K <sub>0</sub> , K <sub>1</sub>	=	Bessel functions of argument ( $\gamma a$ )
i	=	Electron convection current
	=	ac component of electron convection current
J	=	Impressed current density amps/meter
j	=	$\sqrt{-1}$
K	=	$\sqrt{x/B}$
l	=	Length
m	=	Mass of electron
N	=	Wavelengths
P	=	Power
R <sub>s</sub>	=	A-C resistance
r	=	Radius
t	=	Time

# TABLE I. Summary of the results of the calculations.

Calculation	Result
1. $\langle \sigma \rangle$	0.5
2. $\langle \sigma^2 \rangle$	0.25
3. $\langle \sigma^3 \rangle$	0.125
4. $\langle \sigma^4 \rangle$	0.0625
5. $\langle \sigma^5 \rangle$	0.03125
6. $\langle \sigma^6 \rangle$	0.015625
7. $\langle \sigma^7 \rangle$	0.0078125
8. $\langle \sigma^8 \rangle$	0.00390625
9. $\langle \sigma^9 \rangle$	0.001953125
10. $\langle \sigma^{10} \rangle$	0.0009765625
11. $\langle \sigma^{11} \rangle$	0.00048828125
12. $\langle \sigma^{12} \rangle$	0.000244140625
13. $\langle \sigma^{13} \rangle$	0.0001220703125
14. $\langle \sigma^{14} \rangle$	6.103515625e-05
15. $\langle \sigma^{15} \rangle$	3.0517578125e-05
16. $\langle \sigma^{16} \rangle$	1.52587890625e-05
17. $\langle \sigma^{17} \rangle$	7.62939453125e-06
18. $\langle \sigma^{18} \rangle$	3.814697265625e-06
19. $\langle \sigma^{19} \rangle$	1.9073486328125e-06
20. $\langle \sigma^{20} \rangle$	9.5367431640625e-07
21. $\langle \sigma^{21} \rangle$	4.76837158203125e-07
22. $\langle \sigma^{22} \rangle$	2.384185791015625e-07
23. $\langle \sigma^{23} \rangle$	1.1920928955078125e-07
24. $\langle \sigma^{24} \rangle$	5.9604644775390625e-08
25. $\langle \sigma^{25} \rangle$	2.9802322387695312e-08
26. $\langle \sigma^{26} \rangle$	1.4901161193847656e-08
27. $\langle \sigma^{27} \rangle$	7.450580596923828e-09
28. $\langle \sigma^{28} \rangle$	3.725290298461914e-09
29. $\langle \sigma^{29} \rangle$	1.862645149230957e-09
30. $\langle \sigma^{30} \rangle$	9.313225746154785e-10
31. $\langle \sigma^{31} \rangle$	4.656612873077392e-10
32. $\langle \sigma^{32} \rangle$	2.328306436538696e-10
33. $\langle \sigma^{33} \rangle$	1.164153218269348e-10
34. $\langle \sigma^{34} \rangle$	5.82076609134674e-11
35. $\langle \sigma^{35} \rangle$	2.91038304567337e-11
36. $\langle \sigma^{36} \rangle$	1.455191522836685e-11
37. $\langle \sigma^{37} \rangle$	7.275957614183425e-12
38. $\langle \sigma^{38} \rangle$	3.637978807091712e-12
39. $\langle \sigma^{39} \rangle$	1.818989403545856e-12
40. $\langle \sigma^{40} \rangle$	9.09494701772928e-13
41. $\langle \sigma^{41} \rangle$	4.54747350886464e-13
42. $\langle \sigma^{42} \rangle$	2.27373675443232e-13
43. $\langle \sigma^{43} \rangle$	1.13686837721616e-13
44. $\langle \sigma^{44} \rangle$	5.6843418860808e-14
45. $\langle \sigma^{45} \rangle$	2.8421709430404e-14
46. $\langle \sigma^{46} \rangle$	1.4210854715202e-14
47. $\langle \sigma^{47} \rangle$	7.105427357601e-15
48. $\langle \sigma^{48} \rangle$	3.5527136788005e-15
49. $\langle \sigma^{49} \rangle$	1.77635683940025e-15
50. $\langle \sigma^{50} \rangle$	8.88178419700125e-16

$V$	=	Voltage
$V_0$	=	Accelerating voltage to give electron velocity $/U_0$
$v$	=	a-c component of velocity
$x$	=	Shunt reactance
$z$	=	Distance
$\alpha$	=	Attenuation/unit length
$\alpha'$	=	Attenuation/guide wavelength
$\beta_0$	=	Phase constant $\omega/u_0$
$\Gamma$	=	$\frac{\partial \epsilon}{\partial z}$
$\delta$	=	Loss factor
$\delta$	=	Values of $\Gamma$ as roots of equation
$\epsilon_1$	=	dielectric constant of ceramic
$e$	=	Charge to mass ratio of electron
$\lambda_g$	=	Guide wavelength
$u_0$	=	Average velocity of electrons
$\delta$	=	Incremental change in $\beta_0$
$\rho$	=	a-c component of linear charge density
$\rho_0$	=	$I_0/u_0$
$\omega$	=	Angular frequency



## INTRODUCTION

The Traveling Wave Tube today is not a practical device, neither is it a laboratory curiosity. The effort being directed to the accomplishment of a practical Traveling Wave Amplifying Tube has made tremendous strides from the theoretical investigations toward the usable end product.

At present Stanford University is the leader in the theoretical investigation into all phases of the traveling wave tube field. The Bell System Laboratories, under the leadership of Mr. J. R. Pierce are making investigations leading to the use of this device as a primary component for its microwave link systems. The Armed Forces of the U. S. are so interested in the problems and uses of the traveling wave tubes that there are some twelve industrial concerns working on various types for a multitude of ultimate military and commercial uses.

At present the electrical and mechanical problems of the tubes are occupying the time of a small army of engineers and technicians. Each theoretical investigation must be backed by the practical investigation of the engineer.

The advantages of the traveling wave tube for many problems facing the electronic engineer are many. First the traveling wave tube is a wide band radio frequency amplifier and is capable of amplifying equally all frequencies in a two to one frequency band from about 100 MC to 25000 MC or higher. In other words, radio frequency amplification over this range can be accomplished simultaneously in eight tubes, where the same range using klystrons might take 800. In the microwave communication link systems an amplifier of this type has obvious uses, e.g. transmission of several thousand telephone conversations in one transceiver. Also all levels of





amplification may be accomplished at radio frequencies rather than requiring intermediate frequencies and heterodyning. The uses of traveling wave tubes in electronic countermeasures are, also, obvious. All practical radar frequencies may be amplified in very few tube types.

All power levels are being investigated for traveling wave tubes. Proposals have been made by the engineers to develop tubes furnishing power outputs ranging from microwatts to hundreds of Kilowatts.

One system using traveling wave tubes in a communication link is in practical use in England. This link system uses low noise tubes, intermediate power tubes as drivers and ten watt output tubes.

The present tubes in practical use are long glass tubes which are very inefficient and fragile. Several commercial companies in this country have made similar tubes. The tubes which show the greatest promise are those which are more neatly packaged, which are self-aligning in their magnetic solenoids and which are more efficient.

These packaged tubes, however, have created problems which were not apparent in the glass tubes. Most of the present development effort is concerned with the problem of oscillation suppression.

Since these devices are capable of very high gains in comparatively short physical lengths, certain feed back paths inherent to traveling wave tubes must be eliminated.

One of the methods of eliminating this feed back is by attenuating the feedback voltages to such an extent that they are no longer a problem. One of the attacks of attenuating this feedback is the subject of this paper.





Since no mathematical evaluation of "loss" can be made, it will be treated qualitatively. However, some of the factors leading up to the requirement of this addition of loss can be evaluated. They will be more rigorously treated.



# I

## GENERAL DESCRIPTION OF A TRAVELING WAVE TUBE

The traveling wave tube involves a relatively new philosophical and theoretical approach to the radio frequency amplification problem at ultra high frequencies and microwaves, which take advantage of transit time to accomplish the amplification. The traveling wave tube is a broad band amplifier and has the unique characteristic that the gain is constant and independent of frequency over a very wide bandwidth.

The tube itself consists of three essential parts; viz., matching sections, the helix, and the electron gun. The matching sections couple the input and output radio frequency coaxial lines or wave guide to the helix. The helix is constructed to provide a slow wave structure for the signal, and the electron gun is to provide a direct current electron beam to be used as a source of energy for the amplified signal.

To obtain energy transfer between the electron beam and the electromagnetic wave propagating down the helix, the axial velocity of the beam must be nearly equal to the axial velocity of the electromagnetic wave. The axial velocity of the electromagnetic wave is determined by the pitch and diameter of the helix, and therefore the helix determines the operating voltages of the electron gun.

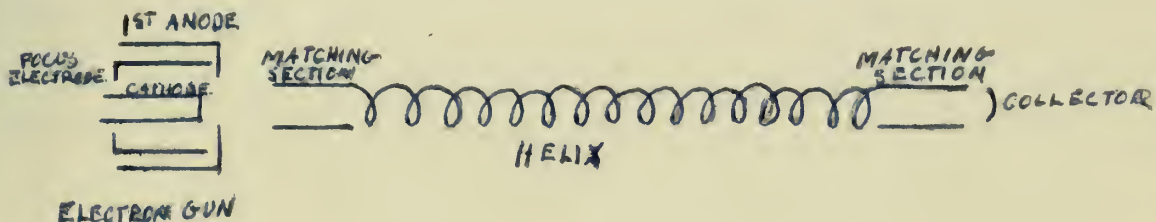


Fig. 1 Traveling wave tube components

# THEORY OF THE ELECTRIC CIRCUIT

Let us consider a circuit consisting of a battery of cells, a variable resistor, and a galvanometer.

The battery consists of  $n$  cells, each having an electromotive force  $E$  and an internal resistance  $r$ . The variable resistor has a resistance  $R$ , and the galvanometer has a resistance  $G$ . The total resistance of the circuit is  $R + G + nr$ . The current  $I$  flowing through the circuit is given by

$$I = \frac{nE}{R + G + nr}$$

Let us suppose that the variable resistor is adjusted so that the current  $I$  is a maximum.

Then the resistance  $R + G$  must be equal to  $nr$ . This is the condition for maximum current.

The maximum current  $I_m$  is then given by

$$I_m = \frac{nE}{2nr}$$

which is independent of the resistance  $G$  of the galvanometer. This is a very important result.

It follows that the maximum current is independent of the resistance  $G$  of the galvanometer.

Let us now suppose that the variable resistor is adjusted so that the current  $I$  is a minimum.

Then the resistance  $R + G$  must be infinite. This is the condition for minimum current.

The minimum current  $I_{\min}$  is then given by

$$I_{\min} = 0$$

which is independent of the resistance  $G$  of the galvanometer. This is also a very important result.

Let us now suppose that the variable resistor is adjusted so that the current  $I$  is a certain value.

Then the resistance  $R + G$  must be a certain value.

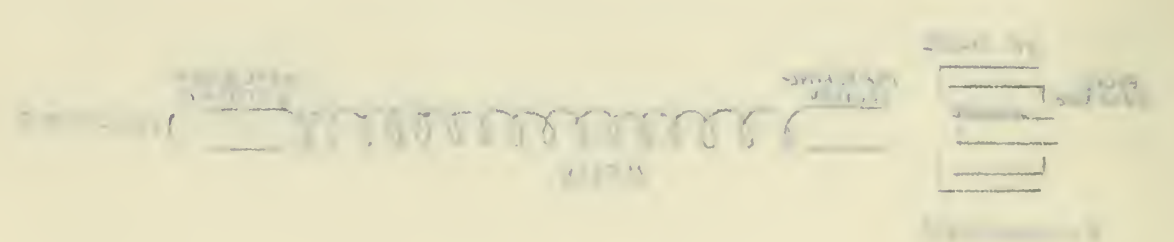


FIG. 1. Circuit for the experiment.

The helical r-f transmission line serves to propagate the radio frequency wave to be amplified and the wave thus propagated travels along the wire at approximately the speed of light. This means that the wave front progresses down the axis of the tube at a rate which is approximately proportional to the ratio of pitch of the helix to the length of a single turn. An electron beam from the electron gun is made to traverse the region inside the helix resulting in an interaction with the axial component of the electromagnetic field in such a manner as to produce several forward moving waves. The resultant wave of interest has negative attenuation or has gain. It extracts energy from the DC beam and grows so that at the output an amplified signal is obtained.

Since the traveling wave tube is a broadband amplifier and gain is practically constant over a wide frequency band the tube may be used to amplify several frequencies several hundred megacycles apart. The tube may, also, be used at any number of frequencies without the necessity of employing tuning controls.

A number of different types of slow wave structures have been developed, and experiments and investigations are in progress in the development of these into practical tubes. An example is the periodic-waveguide traveling wave tube consisting of a slow wave structure made up of washer type laminations inside of a metal sleeve. This structure will reduce the velocity of the electromagnetic waves, and an electron beam may be sent down the center of the structure through the holes in the laminations. This type of tube is a more restricted type and has a much narrower band width than



The first of these is the fact that the United States is a young nation. It has only been about a century and a half since it was founded. This is a very short time in the history of the world. The second is the fact that the United States is a large nation. It covers a vast area of land, and has a large population. This gives it a great advantage in the world. The third is the fact that the United States is a free nation. It has a free press, a free market, and a free government. This gives it a great advantage in the world. The fourth is the fact that the United States is a powerful nation. It has a strong military, a strong economy, and a strong culture. This gives it a great advantage in the world. The fifth is the fact that the United States is a democratic nation. It has a system of government that is based on the principles of democracy. This gives it a great advantage in the world.

The sixth is the fact that the United States is a nation of immigrants. It has a large number of people who have come from other parts of the world. This gives it a great advantage in the world. The seventh is the fact that the United States is a nation of pioneers. It has a long history of exploration and discovery. This gives it a great advantage in the world. The eighth is the fact that the United States is a nation of inventors. It has a long history of innovation and invention. This gives it a great advantage in the world. The ninth is the fact that the United States is a nation of leaders. It has a long history of leadership in the world. This gives it a great advantage in the world. The tenth is the fact that the United States is a nation of heroes. It has a long history of heroism and courage. This gives it a great advantage in the world.

The eleventh is the fact that the United States is a nation of freedom. It has a long history of freedom and liberty. This gives it a great advantage in the world. The twelfth is the fact that the United States is a nation of justice. It has a long history of justice and fairness. This gives it a great advantage in the world. The thirteenth is the fact that the United States is a nation of peace. It has a long history of peace and harmony. This gives it a great advantage in the world. The fourteenth is the fact that the United States is a nation of love. It has a long history of love and compassion. This gives it a great advantage in the world. The fifteenth is the fact that the United States is a nation of hope. It has a long history of hope and optimism. This gives it a great advantage in the world. The sixteenth is the fact that the United States is a nation of faith. It has a long history of faith and belief. This gives it a great advantage in the world. The seventeenth is the fact that the United States is a nation of courage. It has a long history of courage and bravery. This gives it a great advantage in the world. The eighteenth is the fact that the United States is a nation of strength. It has a long history of strength and power. This gives it a great advantage in the world. The nineteenth is the fact that the United States is a nation of wisdom. It has a long history of wisdom and knowledge. This gives it a great advantage in the world. The twentieth is the fact that the United States is a nation of goodness. It has a long history of goodness and virtue. This gives it a great advantage in the world.

the helical wire type and it will not be considered further.





## II

### GENERAL DESCRIPTION OF ASSOCIATED EQUIPMENT FOR OPERATION OF A TRAVELING WAVE TUBE

The fundamental operation of the traveling wave tube is the interaction of an electron beam with a radio frequency axial field. In order to contain the electron beam in the narrow margins of the helix or other radio frequency transmission line of constant diameter some manner of focusing must be utilized. Since the transmission line is at some positive DC potential there is a force acting on the electron beam to spread resulting in the electrons impinging on the line. Noise reduction requires that this interception be held to a minimum.

In order to contain the electron beam some arrangement for magnetic focusing is used. This magnetic field must extend the entire length of helix, requiring several hundred gauss of constant magnetic field strength over a distance of as much as 12 to 15 inches or more. A uniform field of some 300 gauss is required for one low noise application. Iron encased solenoids are in general use for this purpose. A typical coil is four inches in diameter, twelve inches long, weighs thirty pounds and requires fifty watts of power.<sup>(1)</sup> The utilization of permanent magnets for this purpose is now under study.

Other associated equipments are the various power supplies needed for filament power, the various direct current sources for the operation of the electron gun and for the solenoid.

THE FUNDAMENTAL PRINCIPLES OF THE THEORY OF THE  
RELATIONSHIP OF THE HUMAN MIND TO THE  
PHYSICAL WORLD

The fundamental principle of the theory of the relationship of the human mind to the physical world is that the mind is a product of the physical world, and that the physical world is a product of the mind. This principle is the basis of the theory of the relationship of the human mind to the physical world, and it is the basis of the theory of the relationship of the human mind to the physical world. The theory of the relationship of the human mind to the physical world is a theory of the relationship of the human mind to the physical world, and it is a theory of the relationship of the human mind to the physical world. The theory of the relationship of the human mind to the physical world is a theory of the relationship of the human mind to the physical world, and it is a theory of the relationship of the human mind to the physical world.

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### III

#### DEVELOPMENT OF THE GAIN EQUATION(2)

In order to determine the requirements of adding loss to a traveling wave tube, the characteristics of its gain should be studied. The development of the gain equation leads to that end.

Considering the circuit below,

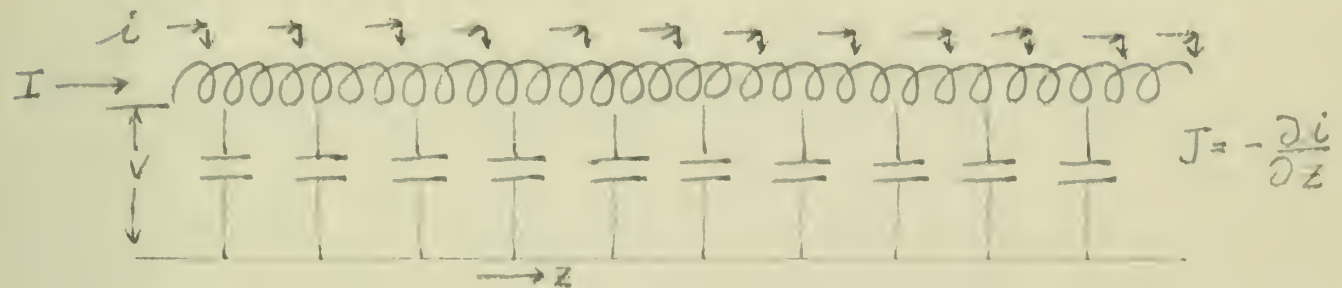


Fig. 2 Equivalent circuit of a traveling wave tube.

$$\frac{\partial I}{\partial z} = -jBV + J \quad 3.1$$

$$\frac{\partial V}{\partial z} = -jXI \quad 3.2$$

Where,  $I$  = current in the line

$z$  = distance

$V$  = voltage impressed

$J$  = impressed current  
amps/meter

$i$  = electron convection current

$B$  = shunt susceptance

$x$  = shunt reactance

$j$  = square root of  
minus one.

$$J = -\frac{\partial i}{\partial z} = \Gamma i \quad 3.3$$

Eq. 3.1 and 3.3 become

$$-\Gamma I = -jBV + \Gamma i \quad 3.4$$

$$-\Gamma V = -jXI \quad 3.5$$

# THEORY OF THE EARTH

The theory of the earth is a branch of geology which deals with the origin and development of the earth and its various parts. It is a science which seeks to explain the causes of the various geological phenomena which we observe in nature.



The earth is composed of several layers. The outermost layer is the crust, which is made of solid rock. Below the crust is the mantle, which is made of molten rock. At the center of the earth is the core, which is made of molten metal.

Crust	= 1	Mantle	= 2
Mantle	= 2	Core	= 1
Core	= 1	Total	= 4

The theory of the earth is a branch of geology which deals with the origin and development of the earth and its various parts. It is a science which seeks to explain the causes of the various geological phenomena which we observe in nature.

To eliminate I;

$$\begin{aligned}
 \Gamma \left( \frac{\Gamma V}{jX} \right) &= -j B V + \Gamma \dot{e} \\
 -\Gamma^2 V &= -j^2 B V X + j \Gamma X \dot{e} \\
 -(\Gamma^2 V + B V X) &= j \Gamma X \dot{e} \\
 -V(\Gamma^2 + B X) &= j \Gamma X \dot{e}
 \end{aligned}
 \tag{3.6}$$

If there were no impressed current ( $i = 0$ ) the right hand side of 3.6 would equal zero. That is BX is to be chosen such that the phase velocity of the circuit of Fig. 1 is the same as for the circuit of a particular traveling wave tube.  $X/B$  is chosen such that for unity power flow, the longitudinal field acting on the electrons,  $-\frac{\partial V}{\partial z}$  is equal to the true field for that same circuit. It can be expressed in transmission line parameters for an undisturbed line ( $i = 0$  in Eq. 3.6).

For the undisturbed line the propagation constant is

$$\Gamma = j \sqrt{BX} \quad BX = -\Gamma^2
 \tag{3.7}$$

The characteristic impedance of the line

$$K = \sqrt{\frac{X}{B}}
 \tag{3.8}$$

and from 3.7 and 3.8 the series reactance is

$$X = -j K \Gamma
 \tag{3.9}$$

Taking BX in 3.7 and in 3.9 and substituting them in 3.6 gives

$$V = \frac{-\Gamma \Gamma K \dot{e}}{(\Gamma^2 - \Gamma^2)}
 \tag{3.10}$$





Sinusoidal variations in time are assumed.

In other words, knowing the natural propagation constant ( $\Gamma'$ ) of the undisturbed line and the new propagation constant ( $\Gamma$ ), the impedance of the line ( $K$ ), the voltage ( $V$ ) on the line will vary with the electron convection current of the beam ( $i$ ).

The next problem is to find the disturbance produced on the electron by the fields of the line.

Units used in MKS

- $e$  Charge to mass ratio of electron ( $1.759 \times 10^{10}$  coulomb/kg)
- $u_0$  Average velocity of electrons
- $V_0$  Voltage by which electrons are accelerated to give them velocity  $u_0$
- $I_0$  Average electron convection current
- $\rho_0$  Average charge per unit length ( $\rho_0 = -I_0/u_0$ )
- $v$  a-c component of velocity
- $p$  a-c component of linear charge density
- $i$  a-c component of electron convection current

The quantities  $v$ ,  $p$ , and  $i$  are assumed to vary with time and distance as  $e^{j(\omega t - \Gamma z)}$

From Newton's Second Law of Motion ( $F = ma$ ) for an electron in an electric field

$$e \frac{\partial V}{\partial z} = -m \frac{\partial (v + u_0)}{\partial t} \quad 3.11$$

But  $u_0$  is the average or d-c velocity and assumed constant with time so that  $\frac{\partial u_0}{\partial t} = 0$ , so 3.11 becomes

$$\frac{e}{m} \frac{\partial V}{\partial z} = \frac{\partial v}{\partial t} \quad 3.11a$$



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The expression for this total derivative in terms of partial derivatives is

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial z} \cdot \frac{dz}{dt}$$

so that from 3.11a

$$\frac{\partial v}{\partial t} + \frac{\partial v}{\partial z} \cdot \frac{dz}{dt} = \gamma \frac{\partial V}{\partial t} \quad 3.12$$

Since  $\frac{dz}{dt}$  is that instantaneous velocity ( $u + v$ ) 3.12 may be written

$$\frac{\partial v}{\partial t} + \frac{\partial v}{\partial z} (u + v) = \gamma \frac{\partial V}{\partial t} \quad 3.13$$

Making small signal approximations that the a-c velocity  $v$  is small compared with the average velocity  $u$  then  $v$  in the parenthesis in 3.13 will be neglected hence the expression becomes

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial z} = \gamma \frac{\partial V}{\partial t} \quad 3.13a$$

Assuming the quantities involved vary as  $e^{(j\omega - \Gamma'z)}$  becomes

$$(j\omega - u\Gamma')v = -\gamma \Gamma' V \quad 3.14$$

and solving 3.14 for  $v$  gives

$$\begin{aligned} v &= \frac{-\gamma \Gamma' V}{(j\omega - u\Gamma')} \\ &= \frac{-\gamma \Gamma' V}{u \left( j\frac{\omega}{u} - \Gamma' \right)} \\ &= \frac{-\gamma \Gamma' V}{u \left( j\beta_c - \Gamma' \right)} \end{aligned} \quad 3.15$$

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We may think of  $\rho_0$  as the phase constant of a disturbance traveling with velocity equal to the electron velocity.

There is another equation to work with called the equation of conservation of charge. If the convection current changes with distance, charge must accumulate or decrease in any small elementary distance, and in one dimension the relation must be

$$\frac{\partial i}{\partial x} = - \frac{\partial \rho}{\partial t} \quad (j\omega t - \Gamma z) \quad 3.17$$

Again assuming the variations as  $e^{j\omega t - \Gamma z}$  3.17 becomes

$$-\Gamma i = -j\omega \rho$$

$$\rho = -j \frac{\Gamma i}{\omega} \quad 3.18$$

The total convection current is the total velocity times the total charge density or

$$-I_c + i = (-u_0 + v)(\rho_0 + \rho)$$

$$= u_0 \rho_0 + u_0 \rho + v \rho_0 + v \rho \quad 3.19$$

but  $I = u_0 \rho_0$  and assume that  $(v \rho)$  the product of two small quantities may be neglected, then

$$-i = u_0 \rho + v \rho_0 \quad 3.20$$

Substituting  $\rho$  from 3.18 into 3.20

$$-i = u_0 \left( -j \frac{\Gamma i}{\omega} \right) + v \rho_0$$

$$u_0 j \frac{\Gamma i}{\omega} + i = v \rho_0$$

$$i \left( 1 + j \frac{\Gamma u_0}{\omega} \right) = v \rho_0$$

$$i = \frac{\rho_0 v}{1 + j \frac{\Gamma u_0}{\omega}} = \frac{\rho_0 v}{1 + j \frac{\Gamma}{\beta_c}}$$

1. The first part of the paper is devoted to the study of the

properties of the function  $f(x)$  defined by the equation

$f(x) = \int_0^x f(t) dt$  and the function  $g(x)$  defined by the equation

$g(x) = \int_0^x g(t) dt$  and the function  $h(x)$  defined by the equation

$h(x) = \int_0^x h(t) dt$  and the function  $k(x)$  defined by the equation

$k(x) = \int_0^x k(t) dt$  and the function  $l(x)$  defined by the equation

2. The

second part of the paper is devoted to the study of the

properties of the function  $m(x)$  defined by the equation

$m(x) = \int_0^x m(t) dt$  and the function  $n(x)$  defined by the equation

$n(x) = \int_0^x n(t) dt$  and the function  $o(x)$  defined by the equation

$o(x) = \int_0^x o(t) dt$  and the function  $p(x)$  defined by the equation

$p(x) = \int_0^x p(t) dt$  and the function  $q(x)$  defined by the equation

$q(x) = \int_0^x q(t) dt$  and the function  $r(x)$  defined by the equation

$r(x) = \int_0^x r(t) dt$  and the function  $s(x)$  defined by the equation

$s(x) = \int_0^x s(t) dt$  and the function  $t(x)$  defined by the equation

$t(x) = \int_0^x t(t) dt$  and the function  $u(x)$  defined by the equation

$u(x) = \int_0^x u(t) dt$  and the function  $v(x)$  defined by the equation

$v(x) = \int_0^x v(t) dt$  and the function  $w(x)$  defined by the equation

$w(x) = \int_0^x w(t) dt$  and the function  $x(x)$  defined by the equation

$x(x) = \int_0^x x(t) dt$  and the function  $y(x)$  defined by the equation

$y(x) = \int_0^x y(t) dt$  and the function  $z(x)$  defined by the equation

$z(x) = \int_0^x z(t) dt$  and the function  $a(x)$  defined by the equation

$a(x) = \int_0^x a(t) dt$  and the function  $b(x)$  defined by the equation

$b(x) = \int_0^x b(t) dt$  and the function  $c(x)$  defined by the equation

$c(x) = \int_0^x c(t) dt$  and the function  $d(x)$  defined by the equation

$d(x) = \int_0^x d(t) dt$  and the function  $e(x)$  defined by the equation

$e(x) = \int_0^x e(t) dt$  and the function  $f(x)$  defined by the equation

Multiplying through by  $\beta_c$  gives

$$i = j \frac{\beta_c I_0 V}{\beta_c - \Gamma'} \quad 3.21$$

Now substituting (v) in 3.15 into 3.21,

$$\begin{aligned} i &= j \frac{\beta_c I_0 V}{\beta_c - \Gamma'} \\ &= j \frac{\beta_c I_0 \frac{V}{V_0} \frac{V_0}{\beta_c - \Gamma'}}{\beta_c - \Gamma'} \\ &= j \frac{\beta_c I_0 V}{2 V_0 (\beta_c - \Gamma')^2} \end{aligned} \quad 3.22$$

which is the convection current in terms of voltage.

Now in Eq. 3.10 the voltage is expressed in terms of the convection current and in 3.22 the convection current is expressed in terms of voltage. Any value of  $\Gamma'$  for which both of these equations are satisfied represents a natural mode of propagation along the circuit and the electron beam.

Combining 3.10 and 3.22

$$\begin{aligned} V &= - \frac{\Gamma' \Gamma'' K}{(\Gamma'^2 - \Gamma''^2)} \cdot j \frac{I_0 \beta_c \Gamma' V}{2 V_0 (\beta_c - \Gamma')^2} \\ 1 &= j \frac{K I_0 \beta_c \Gamma'^2}{2 V_0 (\Gamma'^2 - \Gamma''^2) (\beta_c - \Gamma')^2} \end{aligned} \quad 3.23$$

Equation 3.23 is of the fourth degree in  $\Gamma'$ . Four boundary conditions must be satisfied with the combination of circuit and electron stream. These may be taken as the voltage at the two ends of the helix, the a-c velocity, and the a-c convection current of the electron stream at the point where the electrons are injected.

Since fourth degree equations are not solvable in the general case, but third order algebraic equations may sometimes be solved reduce 3.23 to the third order. Consider only waves in the direction of electron flow, and



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2006-2007 2007-2008 2008-2009

with velocity near that of the electron stream (only such waves will contribute to the observed gain).

First, assume that the electron speed is made equal to the speed of the wave in the absence of electrons (the circuit wave) such that:

$$-\Gamma = j\beta_e \quad 3.24$$

As we are looking for a wave with about the electron speed, we will assume that the propagation constant differs from  $\beta_e$  by a small amount  $\xi$ , so that

$$\begin{aligned} -\Gamma &= -j\beta_e + \xi \\ &= -\Gamma + \xi \end{aligned} \quad 3.25$$

Substituting 3.24 and 3.25 into 3.23

$$\begin{aligned} 1 &= \frac{jK I_0 \beta_e (j\beta_e - \xi)^2 (j\beta_e)}{2V_0 [-\beta_e^2 - (j\beta_e - \xi)^2] [j\beta_e - j\beta_e + \xi]^2} \\ &= \frac{-K I_0 \beta_e^2 (-\beta_e^2 - 2j\beta_e \xi + \xi^2)}{2V_0 [2\beta_e^2 - \xi^2] [\xi^2]} \end{aligned} \quad 3.26$$

Now assuming  $\xi \ll \beta_e$ , then neglecting the terms involving  $\beta_e \xi$  and  $\xi^2$  in the numerator in comparison to  $\beta_e^2$  and neglecting the terms involving  $\xi^2$  in the denominator in comparison to  $\beta_e^2$  gives

$$\begin{aligned} 1 &= \frac{K I_0 \beta_e^2}{4V_0 \beta_e^2 \xi^3} \\ \xi^3 &= -j\beta_e^3 \frac{K I_0}{4V_0} \end{aligned} \quad 3.27$$

Now for convenience rewrite 3.27 in terms of a parameter C where

$$C^3 = \frac{K I_0}{4V_0} \quad 3.28$$

and 3.27 becomes



the following facts (see also Table 1) will be sufficient to

show that the system is

in a state of equilibrium.

The first fact is that the system is

in a state of equilibrium.

The second fact is that the system is

in a state of equilibrium.

The third fact is that the system is

in a state of equilibrium.

The fourth fact is that the system is

in a state of equilibrium.

The fifth fact is that the system is

in a state of equilibrium.

The sixth fact is that the system is

in a state of equilibrium.

The seventh fact is that the system is

in a state of equilibrium.

The eighth fact is that the system is

in a state of equilibrium.

The ninth fact is that the system is

in a state of equilibrium.

The tenth fact is that the system is

in a state of equilibrium.

The eleventh fact is that the system is

in a state of equilibrium.

The twelfth fact is that the system is

Also in place of  $\xi$  introduce a new parameter  $\delta$  so that

$$\xi = \sqrt[3]{C} \delta \quad 3.29$$

Now Eq. 3.27 becomes

$$\begin{aligned} \delta^3 C^3 \delta^3 &= -j \sqrt[3]{C} C^3 \\ \delta^6 &= -j \\ \delta &= (-j)^{1/6} = [e^{j(2n - 1/2)\pi}]^{1/6} \end{aligned} \quad 3.30$$

This equation 3.30 has three roots and represents the three forward moving electromagnetic waves. These roots are  $\delta_1$ ,  $\delta_2$ , and  $\delta_3$ .


$$\begin{aligned} \delta_1 &= e^{-j\pi/12} = \sqrt{3}/2 - j/2 \\ \delta_2 &= e^{-j5\pi/12} = -\sqrt{3}/2 - j/2 \\ \delta_3 &= e^{j\pi/2} = j \end{aligned} \quad 3.31$$


Figure 3. Phase relation of forward moving waves.

Figure 3 shows the three values expressed in 3.31. Equation 3.23 was of the fourth degree and we see that a wave is missing. The missing root was eliminated by the approximations made, which are valid for forward waves only. The fourth root may be found by making separate approximations. The results of such calculations will be expressed as

$$-1^7 = j \beta e \left(1 - \frac{C^3}{4}\right) \quad 3.32$$

As  $C$  is a small quantity, and  $C^3$  even smaller, the backward wave as expressed

July 10, 1891

Dear Sir,

I have the honor to acknowledge the receipt of your letter of the 7th inst.

and in reply to inform you that the same has been forwarded to the proper authorities for their consideration.

I am, Sir, very respectfully,  
Your obedient servant,

J. H. [Signature]

I am, Sir, very respectfully,  
Your obedient servant,  
J. H. [Signature]

I am, Sir, very respectfully,  
Your obedient servant,  
J. H. [Signature]

in 3.32 is practically the same as the backward wave in the absence of electrons. This is to be expected. In the forward direction, there is a cumulative interaction between wave and the electrons because both are moving at about the same speed. In the backward direction there is no cumulative action.

The three forward moving waves or modes vary in the z direction as

$$e^{-\Gamma z} = e^{-j\beta_c z} e^{j(\beta_c - \beta)z} \quad 3.33$$

The first wave associated with  $\beta_1$  is an increasing wave which travels a little more slowly than the electron stream.

$$\begin{aligned} V_1 &= V e^{-j\beta_c z} e^{(\beta_1 - \beta)z} \\ &= V e^{-j(\beta_c + \beta)z} e^{\beta_1 C \beta_c z} \end{aligned} \quad 3.34$$

We are particularly interested in the rate at which the increasing wave or mode increases. For this mode

$$\frac{\text{electronic gain}}{\text{unit length}} = 20 \log_{10} e^{\beta_1 C \frac{\omega}{\omega_0} z} \text{ db} \quad 3.35$$

where  $\beta_1 C = \frac{\omega}{\omega_0}$  and for N wavelengths

$$G_e = 20 \log_{10} e^{\beta_1 C (2\pi N)} \text{ db} \quad 3.36$$

and evaluated

$$G_e = 47.3 C N \quad 3.37$$

With a uniform, lossless helix, the net gain is

$$G = A + B C N \quad 3.38$$

where A is the loss associated with the ratio of voltage of the increasing





wave excited to the total applied voltage.

Assuming that  $V_1 = \frac{V}{3}$

$A = 20 \log 1/3 = -.954$  db so that

$$G = -9.54 + 47.3 \text{ CN db} \quad 3.39$$

This equation assumes a lossless circuit. The practical problems of the actual traveling wave tube must be accounted for and Eq. 3.39 modified. Since the actual circuit in the traveling wave tube will have losses due to the finite conductivity of the components and due to the dielectric loading of the helix support structure, Eq. 3.38 may be modified to

$$G = A + BCN + a l \quad 3.40$$

where  $a$  is the attenuation constant (a negative number) of the entire circuit (helix wire and supporting structure) per unit length and  $l$  is the length.  $A$  is the insertion loss and is always negative.  $B$  is a constant theoretically determined above as 47.3.  $C$  is a variable quantity usually on the order of 0.02 for small signal operation (where the beam is not saturated) and  $N$  is the number of guide wavelengths along the helix.

From equation 3.40 any gain is theoretically possible. This equation, however, does not take into account the possibility of reflections or modes associated with  $\delta_2$  and  $\delta_3$  setting up additional modes associated with  $\delta_1$  at the several discontinuities in the practical tube. It has been found that additional loss must be introduced into the helix structure in order to suppress the other modes of propagation and to prevent their getting to the output where reflections may be set up of sufficient magnitude to cause oscillations. In all practical traveling wave tubes so far developed the problem of oscillation, due to reflections, has been the foremost.

[illegible]

#### IV

#### EVALUATION OF ATTENUATION DUE TO HELIX

#### MATERIAL AND SUPPORT STRUCTURE 3

Since the waves propagated down a helix in a traveling wave tube may be reflected or in some manner the backward wave established, the unwanted waves must be attenuated in some manner in order to prevent oscillations.

In this regard the attenuation due to the helix wire and the supporting structure is of interest. In the traveling wave tube presently being built by Federal Telecommunication Laboratory and several other companies, the helix is made of tungsten wire and the helix is supported by three ceramic rods, which are glazed. The glaze is melted so that each turn of the helix wire is supported at each point of tangency by glass support.

Since no rigorous solution to the field problems of a helix has been made, the attenuation due to the conductivity of the helix material may be found for the idealized helically conducting cylinder. The dielectric loss due to ceramic supporting rods may be computed on the basis of empirical data and on the assumption that the fields outside the helix are not distorted by the presence of the rods.

Assuming the helically conducting sheet, the power flow along the helix is given by

$$P = P_0 e^{-\alpha z}$$

4 - 1

where  $\alpha$  is the attenuation per unit length (in nepers),  $z$  is distance measured from input end along the axis, and  $P$  is the initial power flow. Then,





$$\begin{aligned} \frac{1}{\rho} \frac{d\rho}{dz} &= -2\alpha z \\ \frac{1}{\rho} \frac{d\rho}{dz} &= -2\alpha z \\ \alpha &= -\frac{1}{2\rho} \frac{d\rho}{dz} \end{aligned}$$

4 - 2

Now, if E and H are the electric and magnetic fields, respectively,  $\vec{n}$  is a unit vector normal to the surface of the sheet, and  $R_s$  is the skin effect resistance. Thus

$$\vec{E} = R_s (\vec{n} \times \vec{H}) \quad 4 - 3$$

$$E_z = R_s H_\theta \quad 4 - 4$$

$$E_\theta = -R_s H_z \quad 4 - 5$$

and the power flow per unit area, becomes

$$P_A = \frac{1}{2} (\vec{E} \times \vec{H}) = \frac{1}{2} R_s (|H_\theta|^2 + |H_z|^2) \quad 4 - 6$$

From Pierce "Traveling Wave Tubes" the fields inside the helix are

$$H_\theta = j \frac{B}{K} \frac{\beta_0}{r} I_1(\gamma r) e^{j(\omega t - \beta z)} \quad 4 - 7$$

$$H_z = -j \frac{B}{K} \frac{\gamma}{\beta_0} \frac{I_0(\gamma a)}{I_1(\gamma a)} \frac{1}{\cot \psi} I_0(\gamma r) e^{j(\omega t - \beta z)} \quad 4 - 8$$

and the power loss per unit area from inside becomes,

$$P_{A1} = \frac{R_s}{2} \frac{B^2}{K^2} \left[ \left( \frac{\beta_0}{r} \right)^2 I_1^2(\gamma a) + \left( \frac{\gamma}{\beta_0} \right)^2 \frac{I_0^2(\gamma a)}{I_1^2(\gamma a)} \frac{1}{\cot^2 \psi} \right] \quad 4 - 9$$

Outside the helix from Pierce "Traveling Wave Tubes."

$$H_\theta = -j \frac{B}{K} \frac{\beta_0}{r} \frac{I_0(\gamma a)}{K_0(\gamma a)} K_1(\gamma r) e^{j(\omega t - \beta z)} \quad 4 - 10$$



7-6  
 (1) The first part of the problem is to find the value of  $x$  which satisfies the equation  $x^2 + 2x - 3 = 0$ . This can be done by factoring the quadratic expression on the left-hand side of the equation. We have  $x^2 + 2x - 3 = (x+3)(x-1)$ . Therefore, the equation becomes  $(x+3)(x-1) = 0$ . This implies that either  $x+3 = 0$  or  $x-1 = 0$ . Solving these two equations, we find that  $x = -3$  or  $x = 1$ . Thus, the solutions to the equation are  $x = -3$  and  $x = 1$ .

7-7  
 (2) The second part of the problem is to find the value of  $y$  which satisfies the equation  $y^2 - 4y + 4 = 0$ . This can be done by factoring the quadratic expression on the left-hand side of the equation. We have  $y^2 - 4y + 4 = (y-2)^2$ . Therefore, the equation becomes  $(y-2)^2 = 0$ . This implies that  $y-2 = 0$ . Solving this equation, we find that  $y = 2$ . Thus, the solution to the equation is  $y = 2$ .

7-8  
 (3) The third part of the problem is to find the value of  $z$  which satisfies the equation  $z^2 + 5z + 6 = 0$ . This can be done by factoring the quadratic expression on the left-hand side of the equation. We have  $z^2 + 5z + 6 = (z+2)(z+3)$ . Therefore, the equation becomes  $(z+2)(z+3) = 0$ . This implies that either  $z+2 = 0$  or  $z+3 = 0$ . Solving these two equations, we find that  $z = -2$  or  $z = -3$ . Thus, the solutions to the equation are  $z = -2$  and  $z = -3$ .

7-9  
 (4) The fourth part of the problem is to find the value of  $w$  which satisfies the equation  $w^2 - 7w + 12 = 0$ . This can be done by factoring the quadratic expression on the left-hand side of the equation. We have  $w^2 - 7w + 12 = (w-3)(w-4)$ . Therefore, the equation becomes  $(w-3)(w-4) = 0$ . This implies that either  $w-3 = 0$  or  $w-4 = 0$ . Solving these two equations, we find that  $w = 3$  or  $w = 4$ . Thus, the solutions to the equation are  $w = 3$  and  $w = 4$ .

7-10  
 (5) The fifth part of the problem is to find the value of  $v$  which satisfies the equation  $v^2 + 8v + 15 = 0$ . This can be done by factoring the quadratic expression on the left-hand side of the equation. We have  $v^2 + 8v + 15 = (v+3)(v+5)$ . Therefore, the equation becomes  $(v+3)(v+5) = 0$ . This implies that either  $v+3 = 0$  or  $v+5 = 0$ . Solving these two equations, we find that  $v = -3$  or  $v = -5$ . Thus, the solutions to the equation are  $v = -3$  and  $v = -5$ .

7-11  
 (6) The sixth part of the problem is to find the value of  $u$  which satisfies the equation  $u^2 - 9u + 14 = 0$ . This can be done by factoring the quadratic expression on the left-hand side of the equation. We have  $u^2 - 9u + 14 = (u-2)(u-7)$ . Therefore, the equation becomes  $(u-2)(u-7) = 0$ . This implies that either  $u-2 = 0$  or  $u-7 = 0$ . Solving these two equations, we find that  $u = 2$  or  $u = 7$ . Thus, the solutions to the equation are  $u = 2$  and  $u = 7$ .

$$H_z = j \frac{\beta}{k} \frac{\gamma}{\beta_c} \frac{I_c(\gamma a)}{K_1(\gamma a)} \frac{1}{\cot \psi} K_c(\gamma z) e^{j(\omega t - \beta z)}$$

4 - 11

and the power loss per unit area is

$$P_{AC} = \frac{R_s}{2} \frac{\beta^2}{k^2} \left[ \left( \frac{\beta_0}{\gamma} \right)^2 \frac{I_0^2(\gamma a)}{K_0^2(\gamma a)} K_1^2(\gamma a) + \left( \frac{\gamma}{\beta_c} \right)^2 \frac{I_c^2(\gamma a)}{K_1^2(\gamma a)} K_c^2(\gamma a) \frac{1}{\cot^2 \psi} \right]$$

4 - 12

The total loss is

$$\begin{aligned} P_A &= P_{AL} + P_{AO} \\ &= \frac{R_s}{2} \frac{\beta^2}{k^2} \left[ I_1^2 K_0^2 + I_0^2 K_1^2 \right] \left[ \left( \frac{\beta_0}{\gamma} \right)^2 \frac{1}{K_0^2} + \frac{1}{\cot^2 \psi} \left( \frac{\gamma}{\beta_c} \right)^2 \frac{I_0^2}{I_1^2 K_1^2} \right] \end{aligned}$$

4 - 13

where  $I_0$ ,  $I_1$ ,  $K_0$ ,  $K_1$  are the Bessel Functions with argument  $\gamma a$ .

Now, for an incremental length  $dz$ , the power loss  $dP$  is

$$dP = 2 \pi a P_A dz$$

4 - 14

thus

$$\alpha = - \frac{1}{P} \times \pi a P_A$$

4 - 15

Again from Pierce "Traveling Wave Tubes" to evaluate  $P$ ,

$$\left( E_z^2 / \beta^2 \rho \right)^{1/3} = \left( \beta_0 / \beta_c \right)^{1/3} \left( \gamma / \beta \right)^{1/3} F(\gamma a)$$

4 - 16

$$P = E_z^2 \frac{\beta_0 \beta}{\gamma^4} \frac{1}{F^3(\gamma a)}$$

4 - 17

assuming  $\beta = \gamma$

$$\frac{d\alpha}{R_s} = - \frac{F^3(\gamma a)}{120^4 \times 2\pi} \frac{(\gamma a)^4}{\beta_0 a} \left( I_1^2 K_0^2 + I_0^2 K_1^2 \right) \left[ \left( \frac{\beta_0}{\gamma} \right)^2 \frac{1}{K_0^2} + \frac{1}{\cot^2 \psi} \left( \frac{\gamma}{\beta_c} \right)^2 \frac{I_0^2}{I_1^2 K_1^2} \right]$$

4 - 18

and  $\frac{d\alpha}{R_s}$  computed for values of  $(\gamma a)$  at various values of  $\cot \psi$ .

Let  $f(x) = \frac{1}{x^2} = x^{-2}$

10 - 1

Find the first derivative of  $f(x)$

$$f'(x) = \frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$$

10 - 2

Find the second derivative of  $f(x)$

$$f''(x) = \frac{d}{dx} \left( -\frac{2}{x^3} \right) = -2 \cdot \frac{d}{dx} x^{-3} = -2 \cdot (-3x^{-4}) = \frac{6}{x^4}$$

11 - 1

Find the first derivative of  $f(x) = \sin(x)$

Recall that the derivative of  $\sin(x)$  is  $\cos(x)$

$$f'(x) = \cos(x)$$

11 - 2

Find

11 - 3

Find the first derivative of  $f(x) = \cos(x)$

$$f'(x) = -\sin(x)$$

12 - 1

$$f(x) = \sin(x) + \cos(x)$$

12 - 2

Find

$$f'(x) = \frac{d}{dx} (\sin(x) + \cos(x)) = \cos(x) - \sin(x)$$

13 - 1

Find the first derivative of  $f(x) = \tan(x)$

A more universal result may be obtained if a transfer of variable is used,

where

$$\alpha' \equiv \alpha \lambda_g = \alpha \frac{2\pi}{\beta}$$

4 - 19

then  $\alpha'$  is the attenuation in nepers per guide wave length.

$$\begin{aligned} \frac{\alpha'}{R_s} &= \frac{\alpha a}{R_s} \frac{2\pi}{\beta a} \approx \frac{\alpha a}{R_s} \frac{2\pi}{\gamma a} \\ &= \frac{-F^3(\gamma a)}{120^2} \frac{(\gamma a)^2}{\beta_0 a} (I_1^2 K_0^2 + I_0^2 K_1^2) \left[ \left( \frac{\beta_0}{\gamma} \right)^2 \frac{1}{K_0^2} + \frac{1}{\cot^2 \psi} \left( \frac{\gamma}{\beta_0} \right)^2 \frac{I_0^2}{I_1^2 K_1^2} \right] \end{aligned} \quad 4 - 20$$

Now  $R_s = \left( \frac{\mu_0 \omega}{2\sigma} \right)^{1/2}$  where  $\sigma$  is the conductivity of the material,  $\mu_0$  is the permeability of free space, and  $\omega$  is the angular frequency.

With this method, on a particular helix, an attenuation of approximately 2 db was obtained. This value is much lower than that actually measured.

The total attenuation of the helix must take into account the dielectric losses of the helix support rods. Since these are round rods and there is a considerable amount of distortion, only an approximate evaluation of these losses may be made. The fields are assumed undistorted.

If  $\omega$  is the angular frequency,  $\delta$  is the loss factor,  $\epsilon_1$  is the dielectric constant of the ceramic,  $E$  is the electric field, and  $dA$  is the element of cross sectional area of the rod, the power loss per unit length is

$$P_L = \omega \delta \epsilon_1 \int |E|^2 dA \quad 4 - 21$$

From Pierce, the electric field outside the helix is

$$E_z = B \frac{I_0(\gamma a)}{K_0(\gamma a)} K_0(\gamma r) \quad 4 - 22$$

$$E_r = -j B \frac{\beta}{\gamma} \frac{I_0(\gamma a)}{K_0(\gamma a)} K_1(\gamma r) \quad 4 - 23$$

$$E_\theta = -B \frac{I_0(\gamma a)}{K_0(\gamma a)} \frac{1}{\cot \psi} K_1(\gamma r) \quad 4 - 24$$



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For low voltage tubes  $\beta = \gamma$  so

$$\frac{|E|^2}{B^2} = I_0^2(\gamma a) \left[ \frac{\kappa_0^2(\gamma a)}{\kappa_0^2(\gamma a)} + \frac{\kappa_1^2(\gamma a)}{\kappa_0^2(\gamma a)} + \frac{1}{\cot^2 \psi} \cdot \frac{\kappa_1^2(\gamma a)}{\kappa_0^2(\gamma a)} \right] \quad 4 - 25$$

This may be evaluated for various values of  $(\gamma a)$  and  $\cot \psi$  taking into account the theoretical phase velocity and the loading due to the rods.

A numerical integration may be made of the rods.

If a numerical evaluation is made the incremental areas used may be used to evaluate

$$\int |E|^2 dA = |E_1|^2 \Delta A_1 + |E_2|^2 \Delta A_2 + \dots$$

$$P_L = \omega \delta \epsilon_1 \left[ |E_1|^2 \Delta A_1 + |E_2|^2 \Delta A_2 + \dots \right]$$

4 - 26

An evaluation of this in the case of one specific helix structure gave a loss of approximately 12db. Total loss due to conductivity and support rods is 14 db.

The total loss as measured in the same helix was on the order of 15 db, so the correlation of this approximation and the actual loss is apparent.

This evaluation may be made for any designed helix structure and it will be the term (al) in the gain equation. The tube must be designed so that the gain term overcomes this loss as well as give a net gain of the desired amount.

This loss does not reduce the unwanted waves sufficiently to prevent oscillation due to them. Nor does this loss provide enough attenuations to reduce reflections so that they will not contribute to the tendency to oscillate.



to oscillate.

to reduce reflections so that they will not contribute to the tendency

oscillation and be done. For this loss is not enough, attention

This loss does not reduce the unwanted waves sufficiently to prevent

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will be the term (A) in the gain equation. The loss must be designed so

This evaluation may be made for any designed helix structure and it

do, so the evaluation of this evaluation and the actual loss is important.

The total loss as measured in the helix was of the order of 12

support this is in dB.

have a loss of approximately 12dB. Total loss due to conductivity and

the evaluation of this in the case of one helix helix structure

4 - 26

$$\begin{aligned} \Gamma &= \frac{1}{2} \left[ \frac{1}{\Gamma_0} + \frac{1}{\Gamma_1} + \frac{1}{\Gamma_2} + \dots \right] \\ &= \frac{1}{2} \left[ \frac{1}{\Gamma_0} + \frac{1}{\Gamma_1} + \frac{1}{\Gamma_2} + \dots \right] \end{aligned}$$

used to evaluate

If a numerical evaluation is made the mathematical model used may be

a numerical evaluation may be made of the model.

into account the theoretical losses reflected and the loading due to the rods.

This may be evaluated for various values of (X) and not W section

$$\begin{aligned} \frac{1}{\Gamma} &= \frac{1}{\Gamma_0} + \frac{1}{\Gamma_1} + \frac{1}{\Gamma_2} + \dots \\ &= \frac{1}{\Gamma_0} + \frac{1}{\Gamma_1} + \frac{1}{\Gamma_2} + \dots \end{aligned}$$

For the voltage loss  $\Gamma = 8$  so

## V

### THE APPLICATION OF ADDITIONAL LOSS

The reason for the addition of loss to the traveling wave tube is twofold. First it is desirable to attenuate the modes of propagation which do not add to the gain of the tube and secondly the reflected energy or backward wave must be dissipated such that it will not set up additional forward moving waves which might be in the correct phase to contribute to positive feedback, resulting in oscillation.

The accomplishment of this attenuation must be accomplished in a short physical length so as to not add to the length of the tube. The best method of attenuation would be to actually break the helix at a point where the electron beam has been modulated by the input signal. Each side of the break would be terminated in a matched load and an electromagnetic shield between the broken ends would allow only the energy in the beam to be coupled over to the second or output section of the helix. This method could provide complete elimination of the unwanted waves and reflections. This method is being used in some tubes. However, it complicates the manufacture of the tube so an approximation to the severed helix has been attempted by several industrial laboratories.

The approximation to the severed helix takes several forms and must satisfy certain requirements. These requirements are: 4

1. High attenuation per unit length so that it will not add excessively to the tube length.

2. The loss structure must be mechanically strong to permit handling during assembly of the tube and to survive mechanical vibration tests.



3. Must withstand temperatures in excess of  $700^{\circ}\text{C}$  for short periods during outgassing, and have adequate vacuum properties.

4. Operating temperatures may be as high as  $600^{\circ}\text{C}$  and the loss must retain its strength and electrical properties without fatigue over long periods at this temperature.

5. The materials used must not cause deposits on oxide cathodes or on radio frequency windows or connections.

6. Attenuation should be of the same order of magnitude at room temperature as at operating temperatures in order that cold test predict the loss during operation.

7. A suitable transition of characteristic impedance must be provided as part of the loss structure to allow electromagnetic energy to enter it without reflection.

8. The index of refraction and dielectric polarization must be such as not to cause the radio frequency wave to be slowed or refracted.

9. Conductivity on the surface of the loss must be great enough to carry away any charge which would otherwise collect, deflect the beam, and cause oscillation.

10. The loss must not exhibit resonance absorption in the possible operating range of the tube. It must be broad band without frequency sensitivity.

11. The processing of materials and fabrication should be adaptable to production methods and must be reproducible.

At present there is no structure or material which has met all of these conditions.





The absorption of electromagnetic energy is possible in many materials. 5 It is well understood that this absorption is possible in insulating, semi-conducting, conducting, and magnetic materials. Information is available on absorption in gases and liquids, but they are of no practical value in a high vacuum traveling wave tube. In the preceding section a consideration was made of the dielectric absorption due to the support rods of the helix structure, and it was shown that the material treated had a very low loss and it would not be practical as a loss material.

Semiconductors will not be treated because the energy levels of some traveling wave tubes are so small and because of the conductive changes due to temperature.

Conducting substances result in the best loss material, since they cause loss by the  $I^2R$  heating when incident electromagnetic waves induce eddy currents in the material. In metallic substances the conductivity is electronic. 6 There is an optimum resistivity of a conductor required for production of maximum absorption of energy by the resistance to eddy currents in the conductor.

A number of suggested methods have been tried and found to satisfy some of the imposed requirements. The helix supporting structure may be carbonized, or coated with a conducting material, the surface of the helix wire itself may be made more resistive to provide loss by reducing the wire size by etching. The electric fields may be absorbed by various material if the material is placed in contact with the wire or placed very near it. The use of an additional helix of highly resistive wire can be coupled around the gain helix and couple the energy out of the gain helix. Lossy



materials may be sprayed to form a film on the helix structure. An additional electron beam having the correct phase velocity may be used in attenuating the undesirable waves.

In the present traveling wave tubes using glass tubing support for the helix, the lossy coatings are placed either inside or outside the tubing.

Many loss materials have been tested as conductive coatings on glass and the glass itself has been made conductive. Most of these have found limited application and been discarded. Some of these are; a film of stannic oxide applied at a temperature near the melting point of glass, lead glass rendered conductive by reduction in hydrogen, a glaze consisting of glass and a semiconducting spinels, 7,8 glazed zircon ceramic by means of molybdic oxide or ammonium molybdate followed by reduction, and a suspension of graphite sprayed on the support structure.

Through these tests the use of graphite has become the standard loss material. It satisfies more of the conditions necessary than any other material and the use of colloidal graphite as an internal conductive coating in vacuum tubes has long been in use. The liquid dispersion used usually contains a small amount of an organic protective colloid, such as gum arabic or dextrose which helps keep the graphite particles in suspension, and an alkali silicate is usually used to bind the graphite to the glass.

Properties of graphite appear to have the most desirable electrical and vacuum characteristics for use as an absorber of radio frequency energy. Since no other material is still in the solid state at 1000°C it is well suited for use at the temperatures of operation encountered in traveling wave tubes. Also, the resistivity is high and the room temperature





characteristics are similar to those at the operating temperatures. Graphite is classified a metal and its conductivity is electronic. There is no phase transformation in the operating range due to temperature. One problem encountered using graphite in large quantities, is that it is hard to outgas, but when used in thin films is quite satisfactory. For the internal application to traveling wave tubes very pure chemicals must be used and in this application spectroscopically pure graphite is used.

Several ways of classifying loss structures are; those which intercept the fields only, and those which actually come in contact with the helix wire and provide a parallel conductive path with that of the helix. Some of the examples of the field interception type are the tubes built by Stanford University, the Federal Telecommunications Laboratory's type X-190 (those used in the English link system), the Bell Laboratory's tubes. All of these examples are long glass type of tubes with the helix supported by a glass tube. In these tubes the graphite is painted on the outside of the glass and absorbs the unwanted and reflected energies by intercepting the fields which extend out through the glass. The coupled helix type of loss is one which absorbs the energy through field interception. In this type the coupled helix is wound of highly resistive material around or between the turns of the gain helix.

The contact types of loss are those which are mainly being used in the Federal Telecommunication Laboratory tubes. Several types of tubes are now being made which have ceramic rods to support the helix. Forms of wax are made to allow for the forming of the loss section. In these, several turns of the helix are actually in contact with the loss material and a large portion of the loss section just intercepts the fields in order



to match into and out of the loss section. There has been a patent issued to engineers of the Philco Co. for another loss section of this type. It is described as a helix wire in the loss section of much higher resistivity than that in the input and output sections, and the helix supported by three ceramic rods. In the area of the resistive wire the rods are coated with a conductive graphite material. In this manner the Philco people have attempted to match the impedance of the helix as a transmission line. Another type of loss which shows promise is that of taking a helix supported by ceramic rods and spraying it with a conductive graphite suspension so that the graphite is on the rods and on the helix wire. In this type the conductivity of the wire is changed due to the particulates of graphite adding to the surface roughness and due to the sprayed coating shorting out several turns in the area of the greatest concentration.

Some problems are encountered with these various loss types. In the glass tubes it has been found that the graphite must be sprayed on the tubes and not painted on in strips because the match in and out of each strip is very poor, since the radio frequency energy is actually traveling around the helix circumferentially. If the loss material is sprayed in accordance with a bell shaped curve the match will be good because of the impedance transformer action of the less dense material. There is another problem encountered in this type of loss. It is that only a very small amount of loss can be added in this manner. The normal glass tubing is about twenty thousandths of an inch thick. The fields at this distance are very weak and result in very small induced currents. In the Federal type of loss the graphite is lumped in a very short length and only a small amount of it is in contact with the wire. However the match in and





out of this type is very poor because of the drastic change in characteristic impedance at the first point of contact with the wire. The transformer action of the tapered ends is insufficient and the attenuation in the length of the tapered section is not enough to reduce reflections in insignificant value.

Several tubes have been built with molybdenic oxide as loss material and have been found to be excellent. However, the reduction and, as a result, the conductivity, depends on the exhaust time and the reproducibility is very poor.

The two types of loss on the rod supported helices which shows the most promise are the sprayed distributed loss and another type which is of the field interception type which can be used in this type of tube. The match into and out of these types is quite good and the loss can be made as large as necessary; i.e. at least 60 db. In tubes which operate on output power, levels of ten watts or more these loss arrangements will not dissipate the required amount of power, but on low noise tubes or low power tubes will work very satisfactorily.

The field interception type of loss for the three rod supported helix was developed and investigated at Federal Telecommunication Laboratories while I was there on my industrial tour. 9 In this type of loss application, the graphite is sprayed in a tapered form on the rods before the rods are mounted on the helix. Three rods are held together tangent, and the graphite is sprayed on to the optimum area. This will leave an area free from loss material inside the lines of tangency, where the glaze material will not be impaired and where the helix may be glazed to the rods. This





will leave the helix itself free from changes due to shorting out turns thus not show a sharp impedance change to the waves propagated along the helix. Since the loss may be applied to rods of various dimensions it may be brought at close as necessary to the helix wire for effective absorption of the radio frequency fields. The results of this loss type was that a reproducible attenuation of about 50 db could be produced with good matching characteristics.

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